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A Clothoidal Wall Follower

This article is an extension of the *Improved Wall Follower* or partly builds on it. All the calculations made in the *Improved Wall Follower* regarding the distances between the robot and the walls are still valid. The differences made in this article only affect changes in that the robot in the *Improved Wall Follower* has already traveled exact **circle segments**, whereas the robot in this article drives - as the name of this article reveals - **clothoids**.

Path control on straight-line segments with circles as transitions (Bahnsteuerung auf Geradenbahnsegmenten mit Kreisen als Übergänge)

We remember the *course of the movement* - that is the trajectory (= Bahnkurve) - of the robot in the curves from the *Improved Wall Follower*: This consisted of the robot - while its moving along the wall - getting out of this driving the straight line (v≠0, ω=0) and then turning into a corner by going directly into the arc (v≠0, ω≠0). As soon as the corner piece, as a left or right curve, was finished, the robot again moved from the circular arc directly into another straight line piece (v≠0, ω=0) in order to continue its driving along the wall. One such stretch, consisting of a straight line that merges into a circular arc, followed by another straight line leading out of the arc, is known as **Dubins Path**. [https://de.wikipedia.org/wiki/Klothoide]

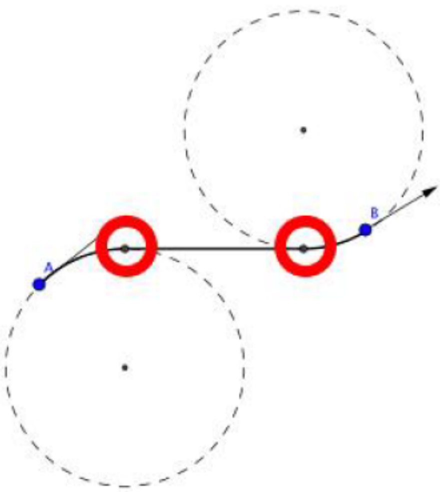

Fig. 1 Dubin's Path

If you were to drive this path with a vehicle in reality, there would be two critical points. These are marked in red in the illustration above. These are the points at which the curvature of the path changes, as it changes from a circular arc to a straight line (and vice versa). These points are critical, because at this point the curvature of the path is changed abruptly. Such a sudden change in the curvature means for the vehicle that at these points, the steering angle must be changed immediately. In reality, however, it is not possible to change the steering angle of a vehicle by any amount within an instant. The vehicle would therefore have to stop at the specified locations to adjust the steering angle to the new curvature. Otherwise, there is a risk that the vehicle will depart from the planned path. In addition, a sudden large change in the steering angle while driving can lead to dangerous situations in traffic. Furthermore, such behavior would be very uncomfortable for potential vehicle passengers, since sudden changes in steering angle can cause unpleasant side jerks. In order to avoid these problems, it is desirable to achieve a **linear instead of a sudden change** in the steering angle at such locations. The goal is to minimize lateral acceleration forces acting in and on the vehicle. So it should be generated a jerk-free driving dynamics, so ideally that in the case of following such a path no back pressure (=Querruck) is felt. This is where the clothoid comes into play.

Path control on straight-line segments with clothoids as transitions

The clothoid is a curve whose curvature is proportional to the distance traveled along the curve. The curvature increases linearly with the arc length of the curve.

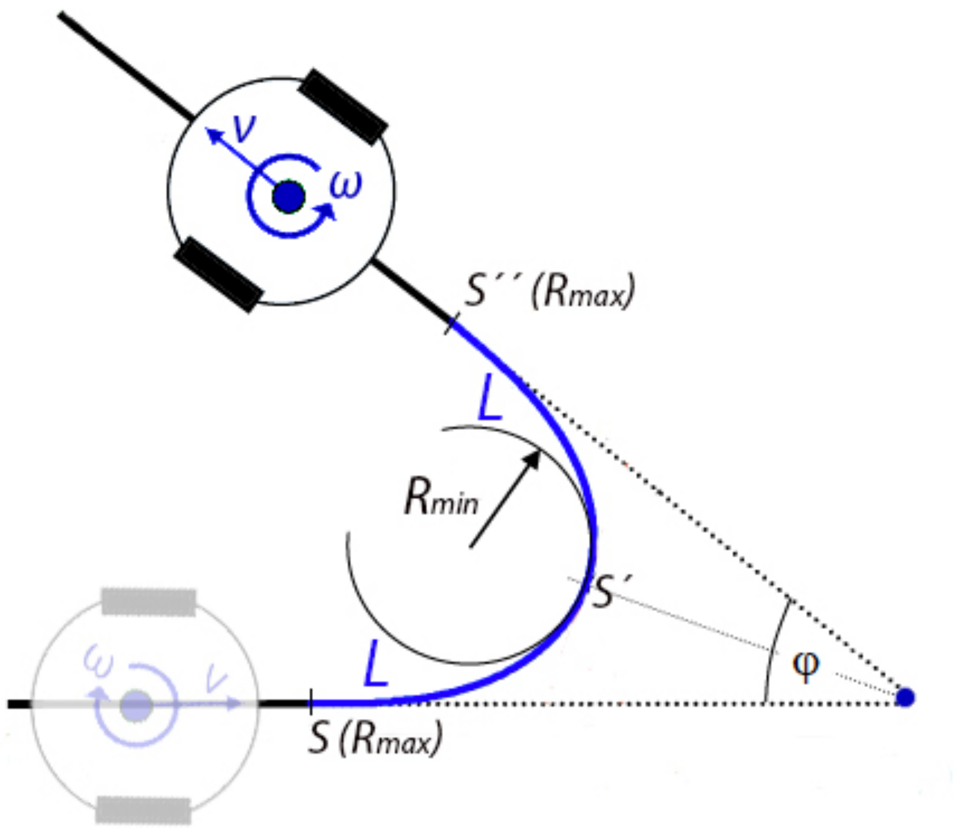

Fig. 2 Robot driving clothoide (when turning, trajectories with continuous curvature are generated)

At the beginning of the clothoid curve (S), the value of the curvature is zero (= straight line) and the radius is the largest (R_{max}). Then the curvature continues to increase up to the middle of the arc (S '), while at the same time the radius gets smaller and smaller (R_{min}). From S 'to S' 'the conditions are reversed, the curvature decreases until the exit of the robot from the curve (S' ') to the same extent as it has risen before, and the radius becomes larger again. This process creates a smooth curve known as the **Euler spiral**.

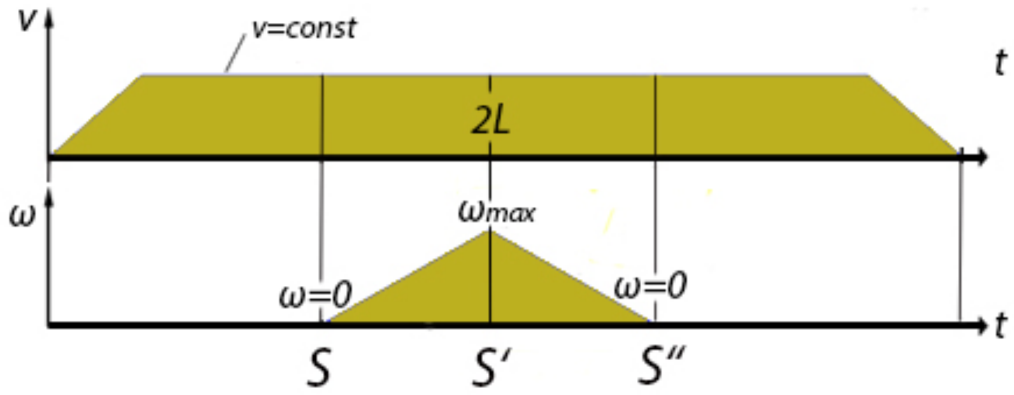

Fig. 3 Dynamic Omega

Such a trajectory is obtained by not setting omega to a fixed value, as was done in the case of driving the circle, but changing omega dynamically, proportionally and steadily. Omega is continuously increased in the path section S-S ', and it is reduced again in the path section S'-S "in the same way.

The clothoid as a trajectory has the advantage over the circle that its curvature increases linearly and thus serves a jerk-free and smooth driving dynamics. Therefore, it is used as a transitional bend in road and railway curves. In the mathematical sense, smoothly means that the curvature of the curve is a continuous function of length. The clothoid thus improves the optical lines of a route. The driver of a vehicle perceives the roadway from a perspective, which, viewed in the direction of travel leads to a significant reduction in longitudinal development. Without a transitional bend, a change of curvature acts like a *kink in the axis* (Knick bzw. Sprung in der Achse). The clothoid as a transitional curve ensures that a curve is better perceived and thus correctly deflected. Even with roller coasters, also rail-bound vehicles, clothoids are used in order not to burden the passengers by strong lateral accelerations (=Querbeschleunigung). In the case of the roller coaster, the speed in each section of the route is known with little variation; thus, the applied transverse forces (=Querkräfte) can be almost completely eliminated by an adapted elevation of the curves. Prerequisite for this are transition bows (=Übergangsbögen). The same applies to the lift supports of cable car installations, in which the roller batteries are often built in the form of a clothoid, in order to offer the passengers of the lift system a higher degree of comfort. [https://de.wikipedia.org/wiki/Klothoide]

Transition from the circle drive to the clothoid drive with the real robot

We now want to exchange the driving of the robot on a pure circular path, as we know it from the *Improved Wall Follower*, by driving on a clothoid. The approach is actually quite simple: To construct a clothoid, we put in each of the two circular paths, which have determined the two cornerings of our robot in the *Improved Wall Follower* - named *radiusCurveLeft* and *radiusCurveRight* - a smaller circle - named *radiusMinLeft* for the left turn and *radiusMinRight* for the right turn. Purely from an intuitive estimation consideration, which looks visually good and seems quite fitting, we conclude that these two radii for the clothoid constructions should be exactly **half the size of their initial radii**.

The following two figures show the relationships:

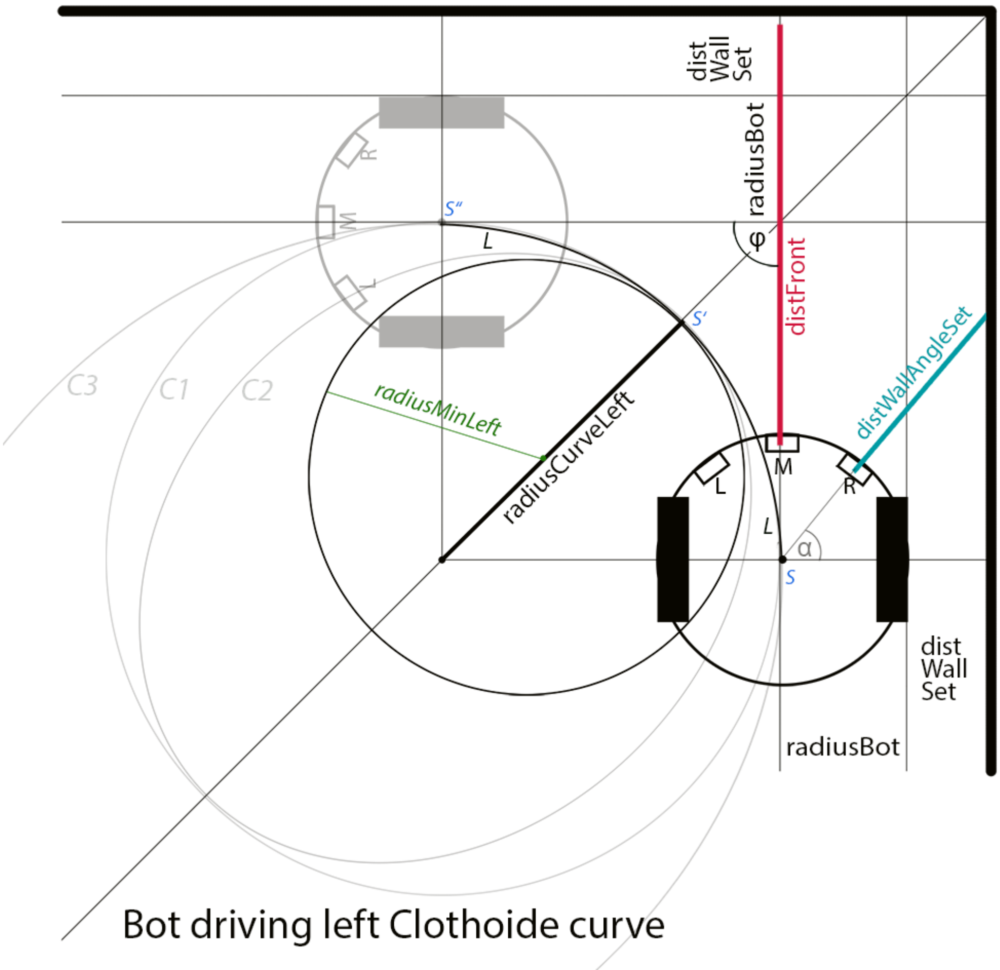

Fig. 4 Bot driving left clothoid

Note: The "clothoids" shown in the two figures are mathematically far from accurate and not true to scale (nicht maßstabsgerecht). *A clothoid has a sinusoidal course. The* *clothoids in the two figures are very simple auxiliary constructions: starting point is circle C1. This is compressed into an ellipse in C2 and enlarged in C3 to such an extent that it passes through the points S and S ".*

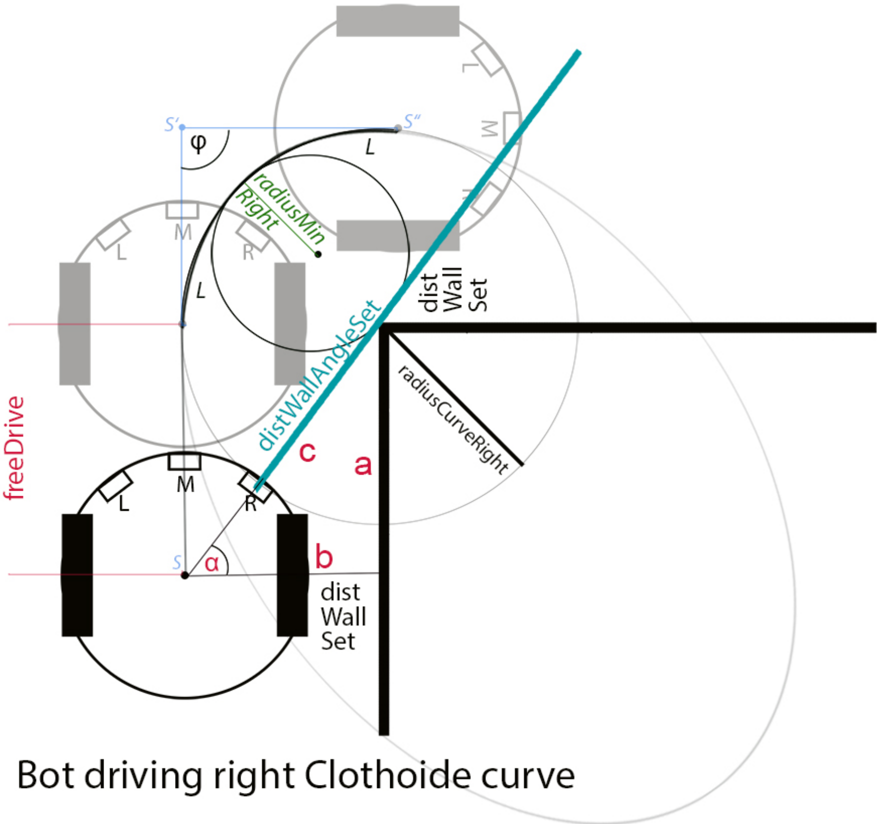

Fig. 5 Bot driving right clothoide

Formula relationships for the left and right curves with clothoid

(Since this article is rather a treatise of great practical relevance, the derivations of the formulas are omitted here; they can be found in the relevant literature[1])

As mentioned above, we have chosen empirically the radii for the clothoid endpoints (*radiusMinRight* and *radiusMinLeft*) half as large as the radii for the clothoid starting points (*radiusCurveRight* and *radiusCurveLeft*):

$$R_{minRight} = 0.5 * radiusCurveRight$$

$$R_{minLeft} = 0.5 * radiusCurveLeft$$

[1] e.g. [1] Path Generation with Clothoid Curve Using Image Processing for Two-Wheel-Drive Autonomous Mobile Robots - Hiroki Ishikawa, Katsuki Noguch, Ryutaro Maki, Haruo Naitoh
[2] Pfadplanung mit stetiger Krümmung - Matthias Barde

The pitch angle (=Steigungswinkel) at the end of the clothoid δ_L is the same for right and left cornering, as we (idealized) assume only right angles for the existences of curves. (If a robot unexpectedly hits another, such as a sharp corner less than 90 degrees, it must use its sensors to pound it out.)

$$\vartheta_L = 90° - 0.5\varphi = 45° = \frac{\pi}{4}$$

As a further value, we need the two maximum rotation speeds ω_{max} for the vOmega interface, which can be calculated from the (fixed) translational velocities $v_{Forward}$ and the minimal radii (see above):

$$R_{min} = \frac{v_{Forward}}{\omega_{max}}$$

$$\omega_{maxRight} = \frac{v_{Forward}}{R_{minRight}}$$

$$\omega_{maxLeft} = \frac{v_{Forward}}{R_{minLeft}}$$

The last value of interest for our robot travel concerns the time duration for driving the clothoid t_c. This is calculated from the length of the clothoid L and the translation speed $v_{Forward}$ of the robot:

$$L = 2 * \vartheta_L * R_{min}$$

$$L_{right} = 2 * \vartheta_L * R_{minRight}$$

$$L_{left} = 2 * \vartheta_L * R_{minLeft}$$

and

$$\mathrm{tcL} = \frac{2L}{\mathrm{vForward}}$$

$$\mathrm{tcL}_{left} = \frac{2L_{left}}{\mathrm{vForward}}$$

$$\mathrm{tcL}_{right} = \frac{2L_{right}}{\mathrm{vForward}}$$

(internal note only: == clotLeftTime, clotRightTime in test-code)

The *total duration times determined here* (=hier ermittelte Gesamtdauerzeiten) now only have to be divided into a suitable number of time segments ($DeltaL_{timeticks}$, $NoL_{timeticks}$) for passing through the clothoid of length L, in which the omega is gradually increased first from 0 to $Omega_{max}$ and then lowered again. The following example shows this process with the following arbitrary values:

ω_{max} = 45°

$DeltaL_{timeticks}$: 1s

(Note: for smooth trajectories this "sampling time" should be sufficiently small, e.g. in the

millisecond range)
(internal note only: == wallFollowerTick in test-code)

tcL: 6s

$$\mathrm{NoL}_{\mathrm{timeticks}} = \frac{tcL}{DeltaL_{timeticks}} = \frac{6s}{1s} = 6$$

(internal note only: == leftCurveTicks, rightCurveTicks in test-code)

$$\omega_{stepWidth} = \frac{\omega_{max}}{NoL_{timeticks}/2} = \frac{45°}{3} = 15°$$

$\mathrm{NoL}_{\mathrm{timetick}}$	0	$1 * \frac{\Delta t}{6}$	$2 * \frac{\Delta t}{6}$	$3 * \frac{\Delta t}{6}$	$4 * \frac{\Delta t}{6}$	$5 * \frac{\Delta t}{6}$	0
Omega ω	0	15°	30°	45°	30°	15°	0

(Note: $\Delta t = DeltaL_{timeticks}$)

Which curve path is shorter for the robot, the circular path or the clothoid?

Of particular academic interest there is the question which path is shorter, the circular path (from the *Improved Wall Follower*) or the clothoid. We want to answer the question by arithmetic.

First the calculation of the length of the clothoid:

$$L = 2 * \vartheta_L * R_{min}$$

$$L = 2 * \frac{\pi}{4} * \frac{1}{2} * radiusCurve = \frac{\pi}{4} * radiusCurve$$

Since the entire clothoid consists of two halves, which is 2L, it follows

$$Curve_{clothoide} == \frac{\pi}{2} * radiusCurve$$

Now the length of the cirular path:

$$Curve_{circle} = \frac{2\pi * radiusCurve}{4} = \frac{\pi}{2} radiusCurve$$

As the two calculations show, in the robot-driven curves for the clothoid and the circle both curves are the same length. As already described in detail above, the difference between the two curves lies in the advantageous, linear curvature course of the clothoid relative to the circle, which serves for a jerk-free driving dynamics.